

Modeling and Control of Well Hydraulics and Drilling Vibrations Mathew Keller, Zheren Ma, Tianheng Feng, Pradeepkumar Ashok, Eric van Oort, Dongmei "Maggie" Chen

Modeling of Well Hydraulics System

Background

- Gas kicks occurs when wellbore pressure < reservoir pressure
- Conventional indicators: increasing pit gain, flow out rate, deviations in pump pressure, etc.
- Conventional well control methods:
 - Stop pump and flow check
 - BOP shut-in
 - Circulate kick out with Driller's or Wait & Weight method \bullet
- Managed pressure drilling (MPD):
 - Increase back-pressure with MPD choke \bullet
 - Constant bottom hole pressure kick circulation

Mathematical Model

- Partial differential equations
 - Two phases: liquid (drilling fluid) and gas (formation or injection)
 - Multiple mass conservation equations for each liquid and gas components
 - One **momentum conservation equation** for the mixture (drift-flux model)

$$\partial_{t}A(x)\begin{bmatrix} \alpha_{l,i}\rho_{l,i} & & & \\ & \ddots & & \\ & \alpha_{g,k}\rho_{g,k} & & \\ & \ddots & \\ & & \ddots & \\ \begin{pmatrix} I \\ \sum_{i=1}^{I} \alpha_{l,i}\rho_{l,i} \end{pmatrix} v_{l} + \begin{pmatrix} K \\ \sum_{k=1}^{K} \alpha_{g,k}\rho_{g,k} \end{pmatrix} v_{g} \end{bmatrix} + \partial_{x}A(x)\begin{bmatrix} \alpha_{l,i}\rho_{l,i} v_{l} & & \\ & \ddots & \\ \begin{pmatrix} I \\ \sum_{i=1}^{I} \alpha_{l,i}\rho_{l,i} \end{pmatrix} v_{l}^{2} + \begin{pmatrix} K \\ \sum_{k=1}^{K} \alpha_{g,k}\rho_{g,k} \end{pmatrix} v_{g}^{2} + p \end{bmatrix} = A(x)\begin{bmatrix} q_{l,i} & & \\ & \ddots & \\ q_{g,k} & & \\ & \ddots & \\ f_{w} + f_{g,k} & & \\ & \ddots & \\ f_{w} + f_{g,k} & & \\ & & \ddots & \\ f_{w} + f_{g,k} & & \\ & & & \end{pmatrix} v_{l}^{2} + \left(\sum_{k=1}^{K} \alpha_{g,k}\rho_{g,k} \right) v_{g}^{2} + p \end{bmatrix}$$

Closure algebraic equations \bullet

Slip law:

$$\begin{cases} v_g = C_0 v_{mix} + v_d \\ v_{mix} = \left(\sum_{i=1}^{l} \alpha_{l,i}\right) v_l + \left(\sum_{k=1}^{K} \alpha_{g,k}\right) v_g \end{cases}$$

Density:

$$\begin{cases} \rho_{l,i} = \rho_{l,i}(p,T) \\ \rho_{g,k} = \rho_{g,k}(p,T) \end{cases}$$

Void fractions:

$$\sum_{i=1}^{l} \alpha_{l,i} + \sum_{k=1}^{K} \alpha_{g,k} = 1$$

- Source term modeling •
 - Non-Newtonian frictional force
 - Yield power law fluid
 - Lee-Gonzalez-Eakin gas viscosity model
 - Two phase choke/bit nozzle model
 - Reservoir model (Production index model)
 - Fracture model (Two-stage model)
 - **Cockrell School of Engineering**

Downward/upward flow

Flow regime dependent

• Linear fluid density model

• Hall-Yarborough real gas

density model







